Types of Mathematical Models

Now that we have an overview of how the mathematical modeling process works, we can ask ourselves some important questions about this process including

- How are mathematical models used in modern day STEM fields?
- What types of models exist in our world and can we categorize these models?
- How and why might I choose to use mathematical models in my career to solve problems I care about?

By exploring answers to these questions, we can begin to form a big-picture view of ways in which mathematical models undergird our built environment. Let's start with a discussion of different categories for the purpose of a mathematical model.

Models to more deeply understand the nature of our physical world

- Hooke's law
- Ohm's law
- Navier-stokes equations
- Heat equation
- Wave equation
- Maxwell's equations
- Schrodinger's equation

Models to conduct unethical or impractical experiments

- Models to study effects of heart surgery on kidney function, drug treatments, laser heart treatment,

Models to inform public policy

- Traffic models
- Cigarette smoke models
- COVID transmission models
- Economic models

Models to make money (i.e. models used to generate revenue)

The major theory that lies at the heart of any for-profit business is quite simple. By definition, profit is the difference between revenues and expenses. To make a profit, a business must sell some good or service at a cost that is higher than what that business pays to generate whatever they sell. The basic model to describe

$$\text{Profit} = \text{Revenues} - \text{Expenses}$$

EXAMPLE 0.0.1

One well-known example a company's effort to generate a mathematical model that increases revenues can be seen in the Netflix prize. In late 2006, the American media company Netflix announced that they would award $1 million to a team who could generate an algorithm that improved the accuracy of the movie recommendation service that Netflix offered to customers. Netflix's recommendation process provides customers with a list of recommended movies based on previous viewing history and movie ratings. Netflix's business model depends on users believing that Netflix is a crucial part of their daily life so that every user is willing to pay the monthly subscription fee. Thus, if Netflix can create a model that provides users accurate recommendations for new movies they might like, the hope is that those users might watch more content and remain loyal subscribers to Netflix services. This is part of how Netflix can increase revenues by increasing retention of its existing customer base. In other words, the mathematical model at the heart of this algorithm was designed to help Netflix generate more revenue to support its daily operations.

EXAMPLE 0.0.2

Options pricing models and model for dynamics of financial markets

Models to save money (i.e. models used to minimize expenses)

EXAMPLE 0.0.3

After the invention of digital computers, larger groups of scientists and mathematicians began to solve the problem of handwriting recognition. This is the problem of accurately translating handwritten symbols into information that a computer can use to collect data and make decisions. While good solutions to this problem have broad applications in many fields of study, in this example we highlight the use of handwritten recognition to decrease the expenses of the United State Postal Service (USPS). In the 1990s, the USPS contracted researchers at the State College of New York to develop handwriting recognition technology. The core of this technology included mathematical models to identify handwriting which the USPS could then use to accurately sort mail. After a decade of work, the USPS put the technology into use to recognize the specific address block and identify city/state combinations as well as street addresses and translate this into an eleven-digit ZIP code. Since the USPS creates a unique 12-digit number to encode every single mailable delivery point in the country (every location where it is willing and able to drop off mail), Automating this process of accurately identifying the 11-digit zip saved the USPS millions of dollars per year in labor costs and increased efficiency in the system.

EXAMPLE 0.0.4

A famous method for solving differential equations using computers is known as the finite element method. Let's study the use of this method within CAD programming in Boeing Company in the 1980s

EXAMPLE 0.0.5

Let's study the use of SPICE software to check electronic circuit designs.

EXAMPLE 0.0.6

George Dantzig's famous simplex algorithm applied to the Berlin Airlift problem.

How linear algebra is used in these models

Now that we have a general overview of the seven steps of the mathematical modeling process, we're going to explore a high-level overview of exactly where the theory of linear algebra fits into this framework. Notice that in Step 3 of our process, modelers transform the real-world problem into an ideal mathematical model. As mentioned above, linear algebra is the study of a collection of ideal mathematical models that come in special forms. In this introductory textbook, we study five types of ideal models, each one having special sub-types. That is the exact subject of our next lesson.

Suggested Exercises:

1. Look over the list of models provided in this section. Choose 1 that relates to your future career and academic interests. Then, put a timer on for 40 minutes. In that time, do some deep reading on that model.